Discrete Listwise Personalized Ranking for Fast Top-N Recommendation with Implicit Feedback

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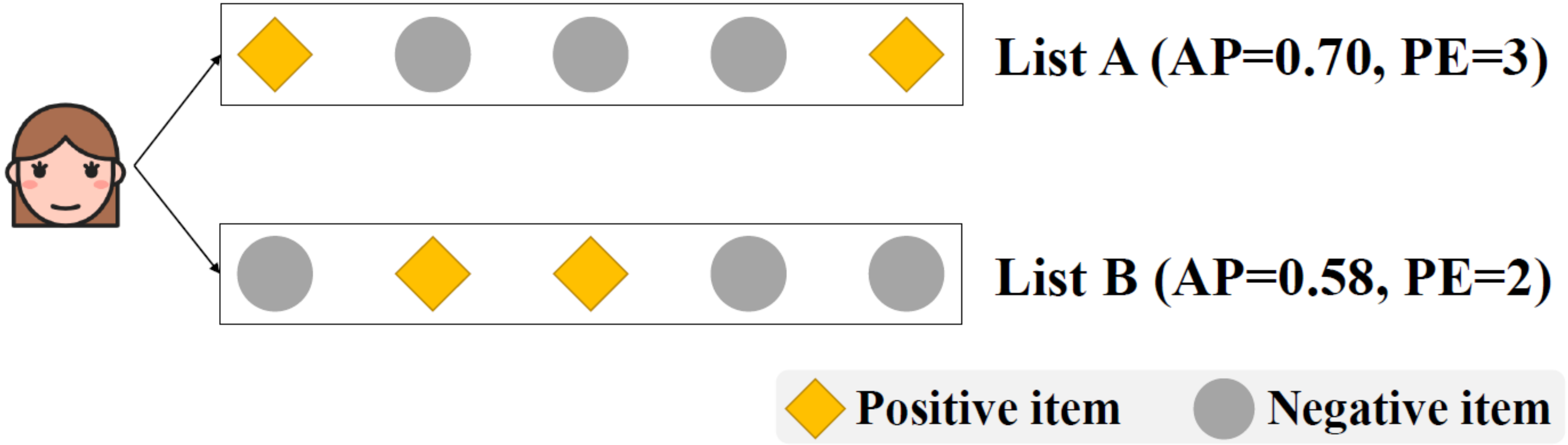
• Outline

- ☐ **Motivation**
  - ☐ Inconsistency Issue in Hashing-based Recommender System
- ☐ **Proposed Method: Discrete Listwise Personalized Ranking (DLPR)**
  - ☐ A Learnable AP-oriented Learning Objective
  - ☐ Solution: Alternating Optimization Strategy
- ☐ **Experiments**
- ☐ **Conclusion**

• Motivation

Current hashing methods for top-N recommendation fail to align their learning objectives (such as pointwise or pairwise loss) with the benchmark metrics for ranking quality (e.g. Average Precision, AP), resulting in sub-optimal accuracy.



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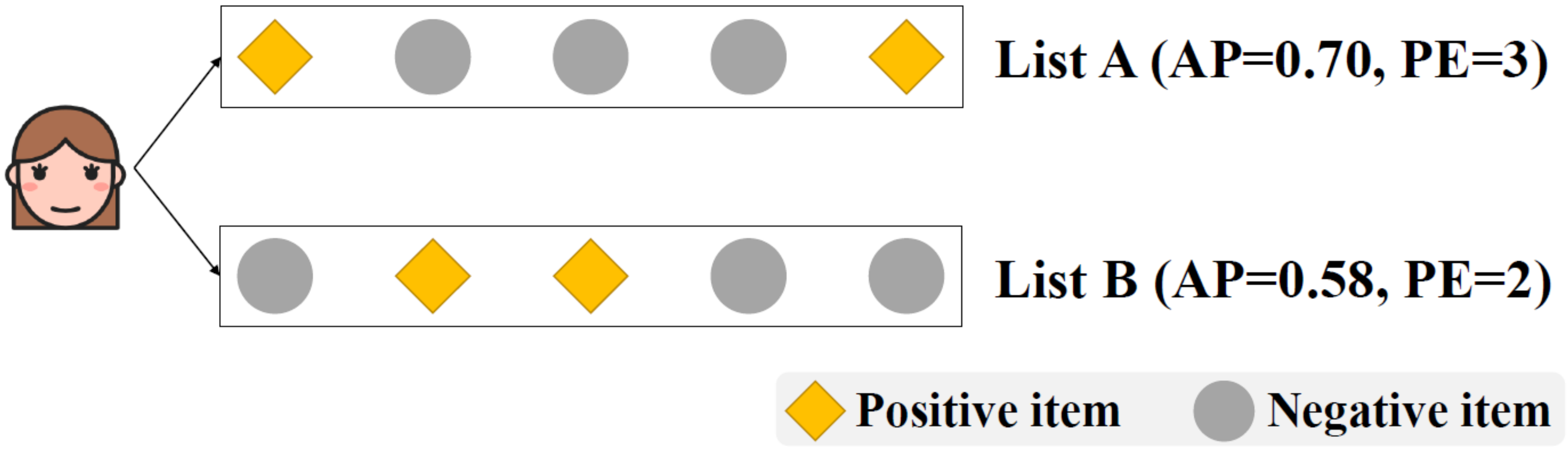

• Discrete Listwise Personalized Ranking (DLPR)

Learning Objective

$$\mathcal{L}(\mathbf{B},\mathbf{D}) = 1 - \frac{1}{m}\sum_{u=1}^{m} AP_u@K$$

$$= 1 - \frac{1}{m}\sum_{u=1}^{m} \frac{\sum_{k=1}^{K} \frac{y_{u\pi_{\hat{\mathbf{y}}_u}(k)}}{k} \sum_{i=1}^{k} y_{u\pi_{\hat{\mathbf{y}}_u}(i)}}{\sum_{k=1}^{K} y_{u\pi_{\hat{\mathbf{y}}_u}(k)}}$$

$$s.t. \mathbf{B} \in \mathcal{D}_{\mathbf{B}}, \mathcal{D}_{\mathbf{B}} := \{\mathbf{B} : \mathbf{B} \in \{\pm 1\}^{f\times m}\},$$
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Due to discrete constraints, optimizing this objective is a highly challenging task since it is generally NP-hard that involves $O(2^{(m+n)f})$ combinatorial search for the hash codes.

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$\pi_{\hat{\mathbf{y}}_u} = sort(\hat{\mathbf{y}}_u)$ is non-differentiable, which cannot be solved by classical optimization algorithm, e.g., gradient-based optimization algorithm

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A simple illustration to make the objective differentiable

$$[0, 1] \longrightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} [0, 1]^{T} \longrightarrow [1, 0]$$

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$$Z_{i,j}^{(\pi_{\mathbf{s}})} = \begin{cases} 1, & j = argmax[(l+1-2i)\mathbf{s} - \mathbf{A_s 1}]; \\ 0, & otherwise. \end{cases}$$

$$A_s[i,j] = |s_i - s_j|$$

$$\widetilde{\mathbf{Z}}_i^{(\pi_{\mathbf{s},\tau})} = softmax[\tau^{-1}((l+1-2i)\mathbf{s} - \mathbf{A_s 1})],$$

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Learning Objective

$$\mathcal{L}(\mathbf{B},\mathbf{D}) = 1 - \frac{1}{m}\sum_{u=1}^{m} \widetilde{AP}_u@K(\mathbf{y}_u, \pi_{\hat{\mathbf{y}}_u,\tau})$$
$$= 1 - \frac{1}{m}\sum_{u=1}^{m} \frac{\sum_{k=1}^{K} \frac{\widetilde{\mathbf{Z}}_k^{\pi_{\hat{\mathbf{y}}_u,\tau}}\mathbf{y}_u}{k} \sum_{i=1}^{k} \widetilde{\mathbf{Z}}_i^{\pi_{\hat{\mathbf{y}}_u,\tau}}\mathbf{y}_u}{\sum_{k=1}^{K} \widetilde{\mathbf{Z}}_k^{\pi_{\hat{\mathbf{y}}_u,\tau}}\mathbf{y}_u},$$
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Discrete Listwise Personalized Ranking (DLPR)

Solution

- The basic idea of the optimization strategy is transforming the original discrete optimization problem into a solvable concave optimization problem with the relaxed continuous solution space. Based on the Lemma that the optimal solution of a concave function is necessarily located at a boundary of the convex set, we can directly obtain the discrete solutions of original discrete optimization problem in the continuous space.

B-subproblem

$$\mathcal{L}_\varsigma(\mathbf{B}) = \begin{cases} (1-\varsigma)\mathcal{L}(\mathbf{B}) + \varsigma tr(\mathbf{B}^T\mathbf{B}), 1 \geq \varsigma \geq 0; \\ (1+\varsigma)\mathcal{L}(\mathbf{B}) + \varsigma tr(\mathbf{B}^T\mathbf{B}), 0 \geq \varsigma \geq -1, \end{cases}$$
$$s.t., \mathbf{B} \in \mathcal{P}_\mathbf{B}, \mathcal{P}_\mathbf{B} := \{\mathbf{B} : \mathbf{B} \in [-1,1]^{f\times m}\}.$$

while B *not converged* **do**

$$\mathbf{B}^* = \arg\min_{\mathbf{B}^*} tr(\nabla\mathcal{L}_\varsigma(\mathbf{B})^T\mathbf{B}^*), s.t. \mathbf{B}^* \in \mathcal{P}$$
$$\alpha = \arg\min_{\alpha} \mathcal{L}_\varsigma(\mathbf{B}+\alpha(\mathbf{B}^*-\mathbf{B})), s.t. 0 \leq \alpha \leq 1$$
$$\mathbf{B} \leftarrow \mathbf{B} + \alpha(\mathbf{B}^* - \mathbf{B})$$

end

D-subproblem can be solved in a similar way.

• Experiments

☐ Datasets

Datasets	#Users	#Items	#Ratings	Density
CiteULike	5,139	16,980	200,866	0.230%
Amazon	31,058	33,193	1,019,442	0.099%

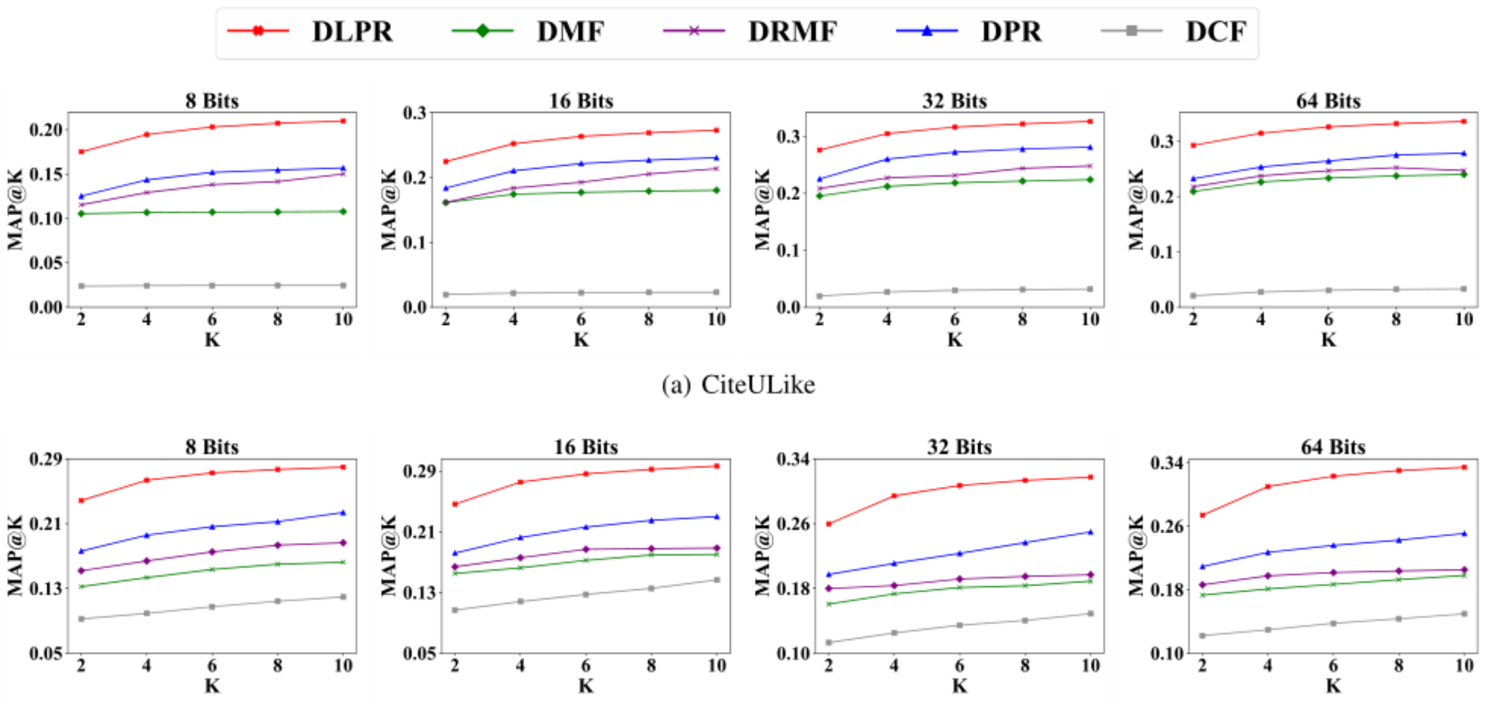
☐ Baselines

DCF [SIGIR’16]
DPR [AAAI’17]
DRMF [KDD’18]
DMF [TKDE’21]

☐ Metrics

MAP, NDCG, Recall, MRR

• Experiments



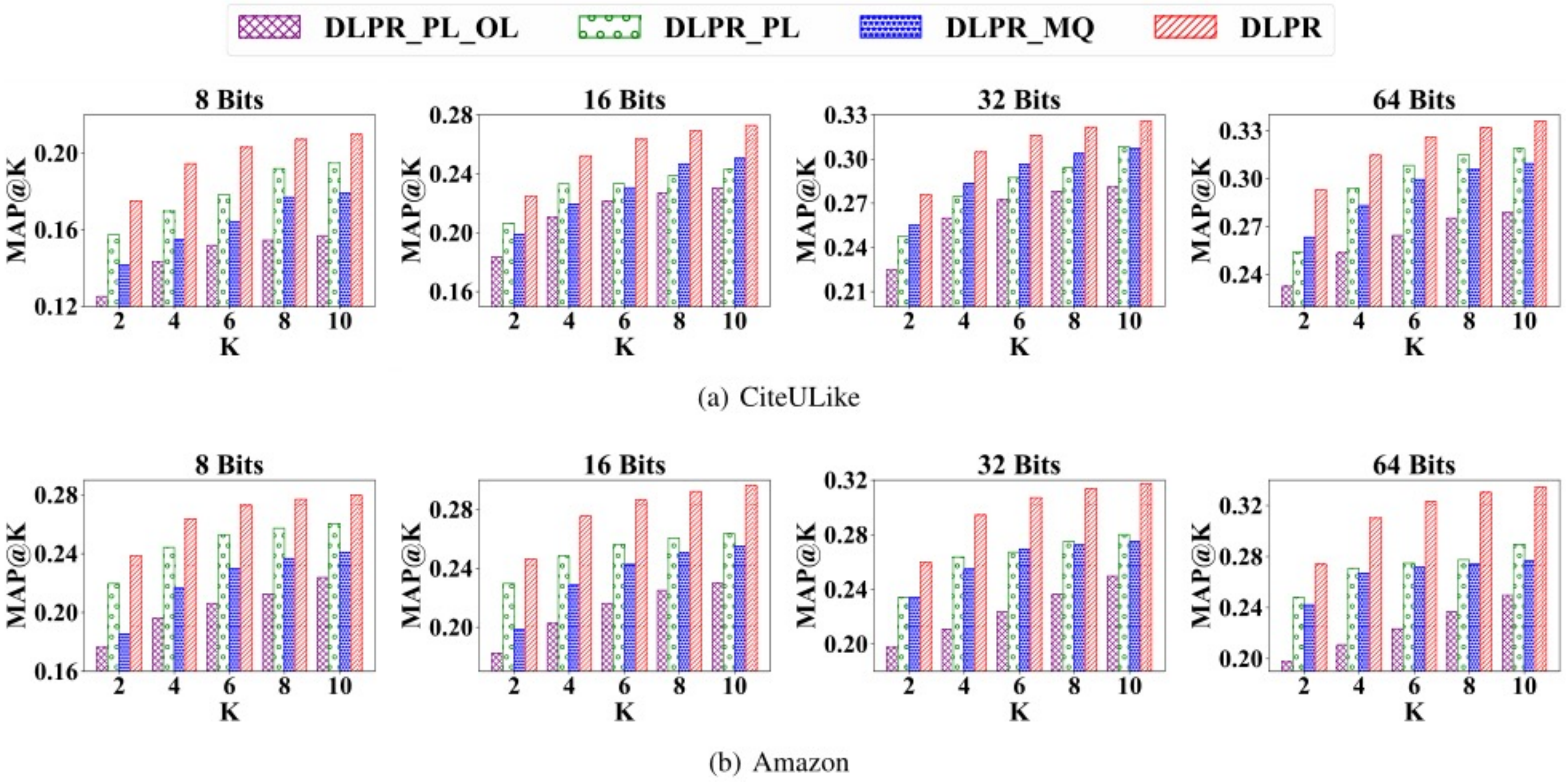
(a) CiteULike

(b) Amazon

Methods	CiteULike			Amazon		
	NDCG@10	Recall@10	MRR	NDCG@10	Recall@10	MRR
DLPR	**0.33975**	**0.56042**	**0.33288**	**0.36042**	**0.61179**	**0.34986**
DMF	0.24090	0.40260	0.25168	0.22047	0.40366	0.23524
DRMF	0.24526	0.44442	0.26233	0.25193	0.42306	0.25379
DPR	0.26950	0.50730	0.28327	0.28131	0.50792	0.29555
DCF	0.03616	0.08620	0.05058	0.18192	0.32291	0.18973

- DLPR, DPR, DRMF > DMF, DCF: listwise and pairwise > pointwise
- DLPR > DPR, DRMF: joint effect of smooth AP loss and optimization algorithm.
- DLPR also obtains higher MRR and Recall than the two pairwise methods: optimizing more information metric, like AP, can boost the performance of the less informative metrics, such as Recall and MRR.

• Experiments



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DLPR-MQ	0.30132	0.53894	0.29183	0.31486	0.54729	0.31055
DLPR-PL	0.31984	0.54882	0.31144	0.32768	0.56656	0.32222
DLPR-PL-OL	0.26950	0.50730	0.28327	0.28131	0.50792	0.29555

- DLPR > DLPR_PL: It verifies that the proposed AP loss is beneficial in binary codes learning.
- DLPR-PL > DLPR-PL-OL: It verifies the superiority of our proposed optimization strategy over the popular one-stage learning paradigm that is widely used in HR.

• Conclusion

✓ Motivated by the fact that current HR schemes fail to align the learning objective with AP metric, we proposed DLPR that is the first personalized ranking method to optimize AP under discrete constraints. Extensive experiments verify the superiority of DLPR over several competitive baselines.

Thank You!